while K_2, K_3, and K_4 would be zero.

Calibration of the bridge may be accomplished by performing preliminary balances with known values of ρ_x and simultaneously solving (15) for K_1 through K_4. A digital computer has proven an invaluable aid in performing this mathematical step. For the rectangular TE_{10} mode, we have calibrated the bridge with three short circuits spaced approximately 120 degrees apart (Fig. 1) followed by a matched termination that had been tuned for zero reflections with a tuned reflectometer [2]. This procedure has yielded accuracies better than 0.001 in magnitude and $\frac{1}{2}$ degree in phase over the entire range $0 \lessapprox |\rho| \lessapprox 1$ at 10 and 24 GHz.

With the circular TE_{01} mode, a well-matched termination is difficult to achieve. We have, therefore, extended the technique to include five preliminary balances: three with short circuits; one with a "matched" termination; and one more with the "matched" termination shifted by approximately one-quarter wave. The resultant five equations were then solved simultaneously for K_1-K_4 and also for the reflection coefficient of the "matched" termination. This procedure has yielded accuracies better than 0.01 in magnitude and 1 degree in phase for $0 \lessapprox |\rho| \lessapprox 1$ at 24, 48, and 70 GHz. We believe that the ultimate accuracy of the circular TE_{01} mode bridges is presently being limited by the modal purity of the available mode transducers.

Differences in the electrical path lengths of reference and sample arms will cause bridge balance to be frequency sensitive. For maximum phase precision, therefore, one should take steps to insure that these paths are nearly equal and that the frequency of the oscillator is stable. Electronic control [8] of the klystron oscillator frequency has proven to be a desirable refinement.

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Magnetodynamic Modes in Axially Magnetized Ferrite Rods Between Two Parallel Conducting Sheets

Open electromagnetic resonators are well-known structures for many applications in the microwave region [1]. Assuming a lossless nonconducting medium partly surrounded by perfectly conducting metal walls, a manifold of undamped oscillations may exist provided the energy of the corresponding electromagnetic field (with finite amplitude) remains finite. In practice, high Q modes occur in open resonators built from isotropic or anisotropic dielectrics [2], [9] or ferrites [3], [4] partly enclosed by well conducting metal walls. In the case of ferrites, the resonator is tunable by a dc magnetic field which may be useful for many technical applications.

In this correspondence, the open resonator shown in Fig. 1 will be investigated. One azimuthally symmetric mode of this resonator structure has been described earlier [4]. All nonradiating electromagnetic eigensolutions of $e^{j\omega t}$ time dependence will be analyzed in the following. The rod is homogeneously magnetized to saturation (saturation magnetization M_S) in z direction by a biasing dc magnetic field $H_0{}^i$. The ferrite is assumed to be lossless; the conductivity of the metal walls shall be perfect. The intensities of the RF electromagnetic fields are supposed to be so small that the linear relation

$$B_f = \mu_0[\mu_1 H_{tf} + j\mu_2(i_z \times H_{tf}) + H_{zf} i_z] \quad (1)$$

between the magnetic induction B and the magnetic field H is valid. In (1)

$$\mu_1 = 1 + \frac{h_0{}^i}{h_0{}^{i2} - w^2}, \qquad \mu_2 = -\frac{w}{h_0{}^{i2} - w^2},$$

where

$$h_0{}^i = \frac{H_0{}^i}{M_S}, \qquad w = \frac{f}{f_m}.$$

Here is

$$f_m = -\frac{\gamma_0}{4\pi} \cdot g \cdot M_S$$

($g = g$ factor, the free electron gyromagnetic ratio $\gamma_0 = -2.21\ 10^7$ cm/A·s $f = \omega/2\pi =$ frequency).

The boundary conditions of the electric field E and the magnetic induction on the metal sheets can be met by

$$E_{tf,a} = F_{tf,a} \sin \beta z, \quad (2)$$

$$E_{zf,a} = F_{zf,a} \cos \beta z, \quad (3)$$

$$H_{tf,a} = G_{tf,a} \cos \beta z, \quad (4)$$

$$H_{zf,a} = G_{zf,a} \sin \beta z, \quad (5)$$

where

$$\beta = \frac{\pi}{l}\rho \qquad (\rho = 0, 1, 2 \cdots). \quad (6)$$

Subscript "f" denotes fields in the ferrite, subscript "a" characterizes fields in the air-filled region. According to Kales [6], differential equations of the z components (subscript z) can be set up from Maxwell's equations. By vector operations, it is possible to derive the transverse components (subscript t) from the z components. The Kales' formulas are modified to

$$F_{zf} = \frac{b}{s_1 - a} u_{1f} + u_{2f}, \quad (7)$$

$$G_{zf} = u_{1f} + \frac{s_2 - a}{b} u_{2f} \quad (8)$$

to get also the solution of an infinitely high dc magnetic field, where the ferrite behaves as an isotropic medium [10]. Here s_1 and s_2 are defined by

$$s_{1,2} = \tfrac{1}{2}[a + c \mp \sqrt{(a-c)^2 + 4bd}], \quad (9)$$

if s_1 has the minus sign and s_2 the plus sign of the square root, respectively. In (9) are

$$a = g^2 - \frac{\mu_2}{\mu_1} k^2, \qquad b = -\omega\mu_0\beta \frac{\mu_2}{\mu_1},$$

$$c = \frac{g^2}{\mu_1}, \qquad d = -\omega\epsilon_0\epsilon_f\beta \frac{\mu_2}{\mu_1},$$

with

$$g^2 = \frac{\omega^2}{c_0{}^2}\epsilon_f\mu_1 - \beta^2, \qquad k^2 = \frac{\omega^2}{c_0{}^2}\epsilon_f\mu_2.$$

(ϵ_f is the relative permittivity of the ferrite material and c_0 the velocity of the light in vacuum.)

The solutions of the differential equations of the z components in cylindrical coordinates are

$$u_{1f} = A_m Z_m(\sigma_1 r) e^{jm\phi}, \quad (10)$$

$$u_{2f} = B_m Z_m(\sigma_2 r) e^{jm\phi}, \quad (11)$$

$$G_{za} = C_m K_m(hr) e^{jm\phi}, \quad (12)$$

$$F_{za} = D_m K_m(hr) e^{jm\phi}, \quad (13)$$

where

$$m = 0, \pm 1, \pm 2, \cdots.$$

In these equations

$$h^2 = \beta^2 - \frac{\omega^2}{c_0{}^2}, \qquad \sigma_{1,2} = \sqrt{|s_{1,2}|},$$

$$Z_m(\sigma_{1,2} r) = J_m(\sigma_{1,2} r) \qquad \text{for } s_{1,2} > 0,$$

$$Z_m(\sigma_{1,2} r) = I_m(\sigma_{1,2} r) \qquad \text{for } s_{1,2} < 0.$$

J_m means the Bessel function of order m, I_m is the modified Bessel function of the first kind and of the order m, K_m is the modified Bessel function of the second kind and of the order m.

Nonradiating eigensolutions only exist for $h^2 > 0$, i.e., z independent nonradiating eigensolutions are not possible. The eigenvalue equation is obtained by meeting the continuity conditions at $r = r_0$

$$A_{11}A_{22} - A_{12}A_{21} = 0. \quad (14a)$$

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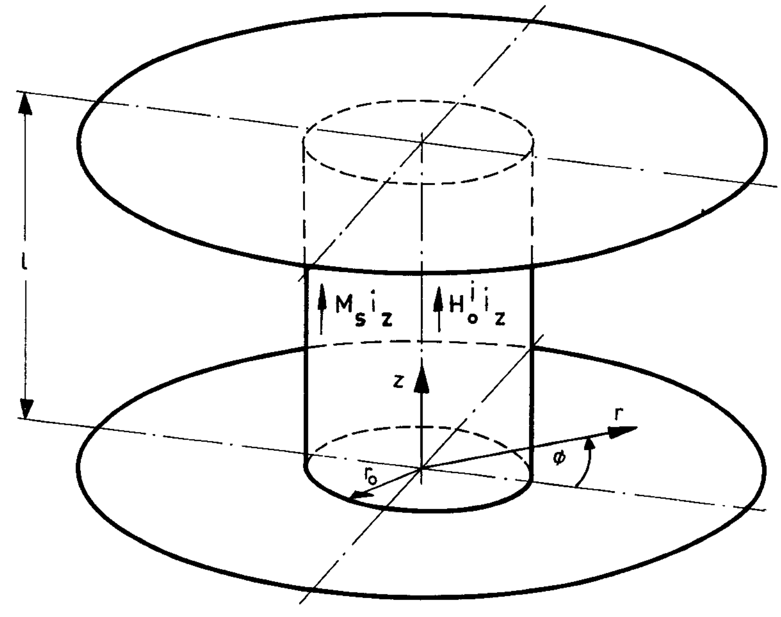

Fig. 1. Cylindrical open resonator.

The abbreviations introduced are

$$A_{11} = Z_m(\sigma_1 r_0)\left\{\frac{\beta m}{h^2 r_0}\frac{b}{s_1 - a} - \frac{\omega\mu_0}{h^2}\frac{K_m'(hr_0)}{K_m(hr_0)} + \frac{mt}{r_0}M_1\right\} - tM_3 Z_m'(\sigma_1 r_0),$$

$$A_{12} = Z_m(\sigma_2 r_0)\left\{\frac{\beta m}{h^2 r_0} - \frac{\omega\mu_0}{h^2}\frac{K_m'(hr_0)}{K_m(hr_0)}\frac{s_2 - a}{b} + \frac{mt}{r_0}M_2\right\} - tM_4 Z_m'(\sigma_2 r_0),$$

$$A_{21} = Z_m(\sigma_1 r_0)\left\{-\frac{\beta m}{h^2 r_0} + \frac{\omega\epsilon_0}{h^2}\frac{K_m'(hr_0)}{K_m(hr_0)}\frac{b}{s_1 - a} - \frac{mt}{r_0}M_5\right\} + tM_7 Z_m'(\sigma_1 r_0),$$

$$A_{22} = Z_m(\sigma_2 r_0)\left\{-\frac{\beta m}{h^2 r_0}\frac{s_2 - a}{b} + \frac{\omega\epsilon_0}{h^2}\frac{K_m'(hr_0)}{K_m(hr_0)} - \frac{mt}{r_0}M_6\right\} + tM_8 Z_m'(\sigma_2 r_0), \qquad (14b)$$

where

$$M_1 = \beta g^2\frac{b}{s_1 - a} + \omega\mu_0\mu_2\beta^2,$$

$$M_2 = \beta g^2 + \omega\mu_0\mu_2\beta^2\frac{s_2 - a}{b},$$

$$M_3 = \beta k^2\frac{b}{s_1 - a} + \omega\mu_0(g^2\mu_1 - k^2\mu_2),$$

$$M_4 = \beta k^2 + \omega\mu_0(g^2\mu_1 - k^2\mu_2)\frac{s_2 - a}{b},$$

$$M_5 = \omega\epsilon_0\epsilon_f k^2\frac{b}{s_1 - a} + \beta g^2,$$

$$M_6 = \omega\epsilon_0\epsilon_f k^2 + \beta g^2\frac{s_2 - a}{b},$$

$$M_7 = \omega\epsilon_0\epsilon_f g^2\frac{b}{s_1 - a} + \beta k^2,$$

$$M_8 = \omega\epsilon_0\epsilon_f g^2 + \beta k^2\frac{s_2 - a}{b},$$

$$\frac{1}{t} = g^4 - k^4.$$

Here Z_m' stands for $\partial Z_m/\partial r$, and K_m' for $\partial K_m/\partial r$, respectively.

If the biasing field is infinitely high ($H_0^i \to \infty$) and the frequencies (w) are finite, the ferrite behaves like an isotropic dielectric material ($\mu_1 = 1$, $\mu_2 = 0$). In this isotropic case (subscript "i"), the following relations are valid

$$g_i^2 = \frac{\omega^2}{c_0^2}\epsilon_f - \beta^2 = a_i = c_i = s_{1i} = s_{2i},$$

$$\frac{1}{t_i} = g_i^4, \qquad M_{2i} = M_{5i} = \beta g_i^2,$$

$$M_{3i} = \omega\mu_0 g_i^2, \qquad M_{8i} = \omega\epsilon_0\epsilon_f g_i^2,$$

$$k_i^2 = b_i = d_i = \frac{b_i}{s_{1i} - a_i} = \frac{s_{2i} - a_i}{b_i}$$

$$= M_{1i} = M_{4i} = M_{6i} = M_{7i} = 0. \qquad (15)$$

The eigenfrequencies of this isotropic dielectric resonator can be calculated from (14) and (15). Results are published in Haas and Godtmann [7]. In the case of the ferrite, the eigenvalue equation (14) has been calculated numerically by computer for various rod dimensions. The rod material was R5, an Mn-Mg-Al ferrite from General Ceramics (U.S.A.). The parameters of the material ($M_S = 1030$ A/cm, g factor $= 1.98$, $\epsilon_f = 11.5$) [8] can be regarded to be independent of the frequency in the interesting region. The h_0^i/w tuning curves of several modes are shown in Fig. 2 for $r_0 = 0.4$ cm and $l = 0.8$ cm.

For $h_0^i \neq \infty$, all nonradiating eigensolutions have z components of the electric field as well as of the magnetic field. The subscripts of the modes are written in the sequence m, n, p; m refers to the ϕ, n to the r, and p to the z direction, respectively. For a finite biasing field, the eigenfrequencies of the $+|m|np$ modes are different from those of the $-|m|np$ modes. The electromagnetic fields of all modes rotate. Looking in the z direction, the whole field configuration of a $+|m|np$ mode rotates counterclockwise, whereas that of a $-|m|np$ mode rotates clockwise ($m \neq 0$). The angular velocity is $w/|m|$.

We will divide the eigensolutions into A (Above) modes and B (Below) modes. The A modes are situated above the curve $\mu_1 = 0$ ($w = \sqrt{h_0^i(h_0^i + 1)}$) in the h_0^i/w plane. The B modes occur below curve $\mu_1 = 0$.

As far as this eigenfrequency is smaller than the radiation frequency (the lowest frequency when radiation starts), the highest eigenfrequency of a nonradiating A mode is the isotropic eigenfrequency ($h_0^i \to \infty$). With decreasing biasing field the eigenfrequency decreases too. The h_0^i/w curve approaches the curve $\mu_1 = 0$. The spectrum of the eigenfrequencies of the A modes is the denser, the closer it is to curve $\mu_1 = 0$.

The isotropic eigenfrequency is common to both the clockwise ($+|m|$) and the counterclockwise ($-|m|$) rotating A modes for the same $|m|$. With decreasing h_0^i, the h_0^i/w curves split up first, and then towards origin of the h_0^i/w plane they meet again. In the case of the isotropic eigenfrequency, the two fields rotating in opposite directions with the same angular velocity and the same amplitudes result in an alternating field. For $m = 0$ we will distinguish between HE_{0np} and EH_{0np} modes. In the case of the isotropic eigenfrequency, the EH_{0np} eigensolution becomes the E_{0np} (or TM_{0np}) mode, whereas the HE_{0np} eigensolution becomes the H_{0np} (or TE_{0np}) mode.

The lowest eigenfrequency of a B mode occurs at $h_0^i \to 0$ (saturation provided). With increasing h_0^i, the eigenfrequency increases too. The h_0^i/w curve stops for the nonradiating eigensolution at the radiation frequency. The h_0^i/w curves of the $HE_{-|m|1p}$ B modes are within the boundaries $w = \sqrt{h_0^i(h_0^i + 1)}$ ($\mu_1 = 0$) and $w = h_0^i + 0.5$, whereas the h_0^i/w curves of the $HE_{-|m|np}$ B modes ($n > 1$) as well as all the other B modes for $h_0^i \to 0$ start at $w > 0.5$. In the case of relatively large rod dimensions, a magnetostatic approximation (similar to the Walker modes [5]) proves a good approximation for the $HE_{-|m|1p}$ B modes.

Part of the h_0^i/w curves calculated was checked by experiments. In Fig. 3, the values measured are marked by crosses, while the corresponding h_0^i/w curves calculated are shown by solid lines. In the experimental resonator, the metal sheets act simultaneously as magnetic pole pieces so that the biasing dc field in the ferrite rod can be considered to be homogeneous. The unloaded Q's varied with the various modes. The maximum values obtained were about 3000. In the case of the A modes, a rapid decrease of the unloaded Q can be observed when the modes enter the (small wave number) spin wave region at $w = h_0^i$. The B modes are always outside this region.

The eigensolutions described herein can be applied to a great number of selective microwave devices, e.g., bandpass filters,

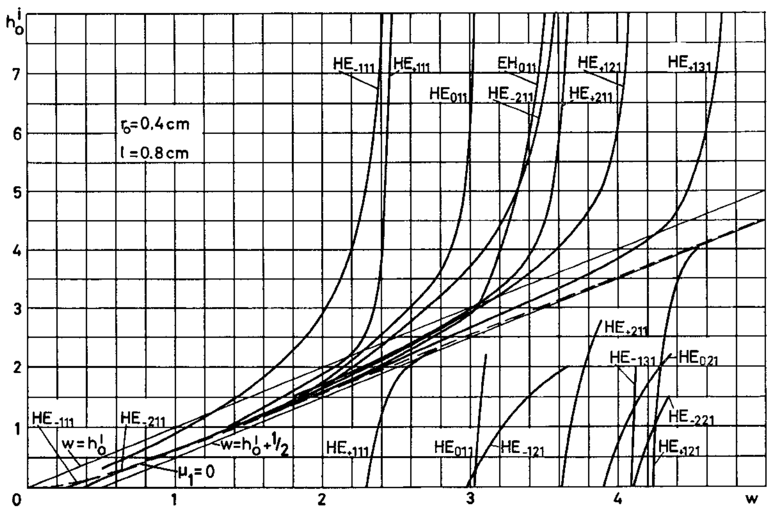

Fig. 2. Calculated h_0^i/w tuning curves of some modes.

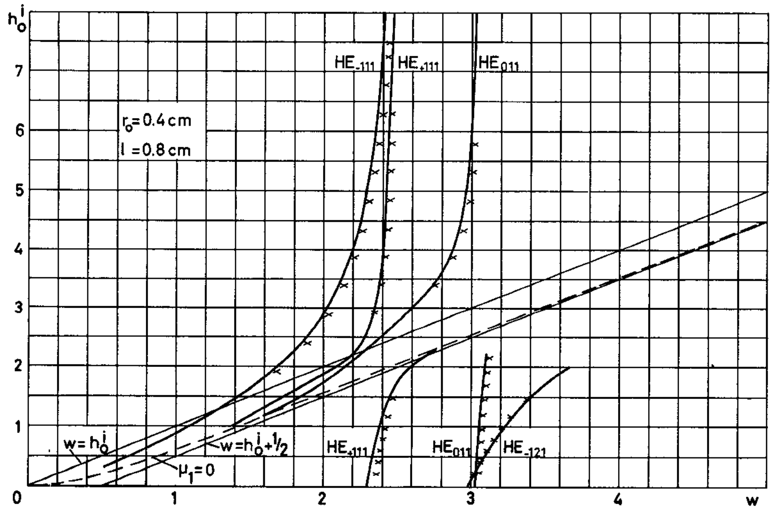

Fig. 3. Experimental results and the corresponding calculated h_0^i/w tuning curves of a few modes.

bandstop filters, and selective modulators with directional characteristic. The magnetodynamic modes are also useful for material measurements especially in the mm-wave region because of the relatively large dimensions.

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Relative Humidity Effects in Microwave Resonators

Several factors affect the frequency stability of microwave resonators. One important factor is the stability of the dielectric medium which fills the resonator. It is the purpose of this correspondence to analyze the effect of the change in dielectric constant upon the resonant frequency of a sealed, air-filled resonator.

Earlier papers have dealt with the effect of the dielectric constant on the propagation of microwaves and its application to the shift of resonant frequency in an open resonator.[1,2] Most of the work is based on an empirical equation which relates the dielectric constant of air to temperature and partial pressures. Since the relationship of the partial pressures is significantly different in open and in closed systems, it is important to analyze the frequency shift in resonators with this in mind.

Montgomery's nomograph shown in Fig. 4, which is the one most frequently referred to on this topic, is most useful for an open resonator operating at high relative humidities and elevated temperatures. Often, however, a resonator is sealed at moderate humidities and room temperature, and completely erroneous results can occur if one tries to extrapolate the results of the nomograph to a closed system. This is because, as the temperature is increased in an open system, the partial pressure of the air increases significantly over the partial pressure of the water vapor, giving a large net increase in dielectric constant and hence a large decrease in resonant frequency.

In a closed system, however, the partial pressure of the air and water vapor increases at a smaller linear rate which is cancelled by the increase in temperature. The net result is that there is a small decrease in dielectric constant which slightly raises the resonant frequency.

A similar situation exists at lower temperatures but is compounded by the fact that, in the closed system, 100 percent relative humidity is soon reached and precipitation of the water vapor occurs with attendent large frequency shift.

In order to analyze the situation, let us consider the equation

$$f\lambda = (\mu\epsilon)^{-1/2}. \tag{1}$$

With the assumptions of Montgomery[1] (i.e., medium does not introduce excessive or variable loss), one can derive

$$\frac{\Delta f}{f} = -\frac{1}{2}\frac{\Delta k_e}{k_e} \tag{2}$$

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